

3.4 Trigonometric Proof

Question Paper

Course	CIEA Level Maths
Section	3. Trigonometry
Topic	3.4 Trigonometric Proof
Difficulty	Hard

Time allowed: 40
Score: /33
Percentage: /100

Question 1

Given the identity

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

prove the following identities:

- (i) $\cos 2\theta \equiv \cos^2 \theta - \sin^2 \theta$
- (ii) $\cos 2\theta \equiv 1 - 2 \sin^2 \theta$
- (iii) $\cos 2\theta \equiv 2 \cos^2 \theta - 1$

[4 marks]

Question 2

(i) Prove the identity

$$\sin 3\theta \equiv 3 \sin \theta - 4 \sin^3 \theta$$

(ii) Show by counter-example that

$$\cos 3\theta \equiv 3 \cos \theta - 4 \cos^3 \theta$$

[5 marks]

Question 3

Show that

$$\cos 4\theta + \cos \frac{\pi}{3} \equiv 8 \sin^4 \theta - 8 \sin^2 \theta + \frac{3}{2}$$

[5 marks]

Question 4

Prove that

$$\cot^2 \theta - \tan^2 \theta \equiv 4 \cot 2\theta \operatorname{cosec} 2\theta$$

[5 marks]

Question 5

Prove the identity

$$\frac{1 - \tan^2 x}{\cos 2x} \equiv \sec^2 x \quad x \neq \frac{2k + 1}{4} \pi$$

[5 marks]

Question 6

Prove the identity

$$\operatorname{cosec} x \equiv \frac{\frac{1}{2} \sec^2 \frac{x}{2}}{\tan \frac{x}{2}}$$

[4 marks]

Question 7

Show that

$$\tan \frac{x}{2} \equiv \frac{1}{\operatorname{cosec} x + \cot x} \quad x \neq 2k\pi$$

[5 marks]

